Reevaluation and replacement of terms in the sampling theory

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***Abstract* – Terms should be short, unique, unambiguous and self-explanatory. Transferring terms from one field of the science to the other should be made with caution. Basic terms of the sampling theory are discussed in the paper. It is shown they are incorrect, misleading or inexact and should be replaced. New terms with definitions and explanations are given for the replacement of the incorrect terms.**

***Keywords* - terms, terminology, sampling theory**

I. NTRODUCTION

Each engineering term should as short as possible, unique and self-explanatory. The term should be correct and leading to exact mathematical presentation if possible. The definition of the term should be a logical expansion of the term and not unexpected explanation. The term should be an abbreviation of the definition or an abridged definition.

The signal sampling theory (SST) as explained and applied in [1-10] contains a lot of terms which should be reevaluated, rejected and replaced with new terms explaining better the nature of the signal sampling and reconstruction process. The paper is dealing with that subject. It is intended to help students, researchers and engineers to clarify the SST.

II. OLD, INCORRECT AND MISLEADING TERMS

The terms below are considered incorrect and misleading from engineering (physical) point of view. Some of them are still acceptable from mathematical or another point of view.

Aliasing - a misleading term meaning in most of the cases that the analog signal (AS) is not adequately sampled and filtered. Also the term is used to show that a sinusoid (or co-sinusoid) is changing from one frequency to another. In the second case it is better to use the "coefficient of the change of the frequency " $K_{ch}= F_{it}/F_{rs}$, where F_{it} is the initial frequency and F_{rs} is the resulting frequency. (The word "alias" has a criminal meaning in [11, 12]).

Classical sampling theorem (CST) – is an oversimplified sampling theorem generally stating that two samples are enough to reconstruct "exactly" band limited signals (BLS) with maximal frequency F_{smax},. According to the CST the sampling rate F_s should be selected according to the equation $F_s = F_{smax}$ or $F_s >= F_{smax}$. The CST is based on Fourier series. It is proved [13] that with CST: 1/ the simplest band limited signal (SBLS) cannot be reconstructed always and 2/ the amplitude errors cannot be evaluated. The theorem is not taking into account the errors due to non synchronized signal sampling and is pretending for "exact" reconstruction. In general the CST is applicable for synchronized sampling with SSF N>=2 without errors evaluation and with low pass filtering.

Co-sine wave (co-sinusoidal signal (CS)) – a simplified but still real (not over simplified) version of the simplest band wide signal (SBLS). Co-sine wave is a SBLS consist with direct current (DC) component with zero amplitude and cosine component with non zero amplitude and usually with zero phase. The cosine wave has four parameters to reconstruct.

Decimation - incorrect term (nothing to do with the number 10), meaning "reducing of the sampling rate by calculations or omissions" especially during the signal reconstruction. (The decimation in the ancient Roman army is giving misleading, non technical and cruel historical background of the term).

Gibbs phenomenon – a non existing phenomenon in the engineering world. It is a mathematical phenomenon when a function with infinite slew rate (rate of change or first derivative) and/or ideal angles is approximated with Fourier series. The phenomenon is often illustrated with ideal (physically impossible) rectangular pulse and wrongly associated with the process of "ringing" due to inappropriate impedance loading. The real signal (RS) is always a "smooth function" and even if it is "truncated" it has: 1/ finite rate of change, 2/ rounded (not ideal, not broken) angles, 3/ finite number of spectral lines and 4/ finite energy in every moment. These basics properties of the real signals are making Gibbs phenomenon non-existing and misleading in the engineering world. Nevertheless it is implemented in the software packages used by engineers as Mathlab.

Nyquist rate, Nyquist frequency – The term means several different things: 1. The highest frequency in the signal spectrum F_{smax}, 2. Twice the maximal frequency in the signal spectrum, 3. The sampling rate which is twice the maximal frequency in the signal spectrum, etc. This multi-definition is cased by the misleading interpretation is one of the proof that the classical SST is not accurate.

Delta function (or unit pulse) - unreal function used to construct the Dirac comb. No practical value in the sampling theory, because is leading to the model "take and forget". In a real system the digital samples are always stored and used. The applicable model is always "take and memorize".

Comb function or **Dirac comb**– non real function representing the sampling model "take and forget", meaning that the sample is not memorized until the next

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sample came. (Related to the “delta function”, “staircase function” and “trapezoidal function”)

Ideal rectangular pulse, ideal triangular pulse and ideal saw tooth pulses – Oversimplified models witch do not respect the basic properties of the real signals. If they are applied they are leading to the Gibbs phenomenon and infinite Fourier series.

Noise – Misleading terms used mainly to represent: 1. Errors during the conversion process. 2. Unwanted signal added to the useful one. 3. Signal which cannot be used but is added to the useful signal. 4. Errors due to calculations.

Dirichlet conditions – conditions wrongly associated with real signals and Fourier series. It should be noted that every real signal is satisfying conditions more strongly that these conditions, and consequently the conditions are not applicable to the signal sampling theory. The Dirichlet conditions are way to say that the Fourier transform and Fourier series are applicable only to real signals, represented by mathematical function and not to all mathematical functions. Every physical signal is: 1. Satisfying the Dirichlet conditions (finite number of maximums, minimums and discontinuities in a given finite interval), 2. Integrable for the time of its existence and 3. Can be represented by a sum of the SBLS.

Dither – a method stating that adding noise to signal could be “good thing”. In fact adding noise is always a ”bad thing” and the method is giving non reproducible results.

First order hold and **zero older hold** – Is it possible to deduce the definition of these terms? The terms “take and memorize (until the next sample come)” , “sample and hold” or “take and forget” are much clearer and self-explanatory.

Fourier series – oversimplified presentation of the real signals as a sum of sine and cosine waves with harmonic frequencies and with zero phases and zero DC components. Mathematically the sum could be infinite. In fact every real signal could be represented as a finite sum of SBLS with not obligatory harmonic sine and cosine components and with not obligatory zero phases and zero DC components.

Over sampling – sampling with frequency higher than Nyquist frequency and meaning to certain “redundancy” which is incorrect. Should be replaced with the terms “Signal sampling factor” (SSF) $N=F_s/F_d$ stating its value

Over sampling and averaging for additional bits of resolution – misleading method for “adding bits” to ADC (beyond its accuracy) with collection of a lot of samples and averaging them. In fact most of the “added bits” have random or not easy reproducible values. The “method” is requiring a lot of memory and computational power and is not efficient and not reliable. In fact the method is a kind of low pass filtering

Resolution – most often the total number of bits used to represent the signal or a function. All of these bits are not obligatory reproducible. In most of the cases the resolution is higher than accuracy and is more or less commercial (non technical and exact) parameter.

Reconstruction of the signal from zero samples - A misleading conception. Obviously there is no way the reconstruct a signal parameter if the information for that parameter is not carried by the samples or in another carrier.

Reconstruction filter – in most of the cases is associated wrongly only with low pass filter. In fact it could be also band pass filter, who is giving the possibility to reconstruct a sine signal even with SSF N<2 in some cases. Also it could be any filter reconstructing the initial signal with the given accuracy in each parameter.

Sine wave (sinusoidal signal (SS)) – a simplified but still real (not over simplified) version of the SBLS**.** It has four parameters to reconstruct. Accepting that the phase and the direct current component are zeros do not suppress them from reconstruction.

Sin (x)/x - artificial function wrongly associated with sampling and reconstruction process of the analog signals. The function is one of the proofs that something artificial is used in the sampling and reconstruction of the signals.

Step function – too idealized transition. If tact should replaced with transition with specified rate of change (slew rate) and defined rounded angles.

Staircase function – Idealized model of the process “take and memorize” with “idealized angles”.

Window – “limitation” of parameter. In some cases it is much clear to use terms “limit”/“limited” and stating the corresponding parameter.

White noise – noise with parameters impossible to generate. The amplitude, spectrum, power and number of spectral lines in the real noise are always limited and does not correspond to the definition of the white noise

III. NEW TERMS

New terms are introduced, defined and listed in alphabetical order. The given definition is self explanatory.

Absolute accuracy (of conversion) – the basic technical term describing the conversion process giving reproducible results. It should be compared with “resolution” and “precision” which are commercial terms and are giving not always repetitive results.

Angle of the first sample θ_0 is the angle between the beginning of the coordinate system (x=y=0 or t=0) of the signal and the moment of the first sample. It is measurable in degrees and is defined especially for a SS, CS and SBLS.

Angle of the maximal deviation from the maximal value of the SS, or the angle of the maximal amplitude error when a SS is sampled (θ_{Emax}). θ_{Emax} for N=> 2 is given with the equation below

$$\theta_{Emax} = 360/(2N) = 180/N \qquad (1)$$

(Law of the) Average amplitude error during the conversion of SS (DC and phase components are zeros) is given with the equation below:

$$E_{max}=1-\cos(90//N) \qquad (2)$$

(Law of the) Average amplitude error during the conversion of CS (DC and phase components are zeros) given with the equation below:

$$E_{max}=1-\sin(90/N) \qquad (3)$$

Basic parameters of the sampling process – The three basis parameters of the idealized but still representative sampling process are: 1/ signal sampling factor (SSF) N, 2/

number of the bits of the converter n (the accuracy of the converter not its resolution) and 3/ angle of the first sample φ_0. The sampling process is fully defined by these three parameters:

Coefficient (factor) of the change of the frequency – K_{ch} is defined as follows:

$$K_{ch}= F_{it}/F_{rs}, \quad (4)$$

where F_{it} is the initial frequency (or a sine or co-sine wave) and F_{rs} is the resulting frequency. K_{ch} is used to show that the frequency of a sine or co-sine wave is changed (usually due to some non linear process and filtering).

Factor of the sample and hold circuits $F_{s/h}$. Term describing the effectiveness of adding a sample and hold circuit. It should be greater than 1 to show increasing performance. If it $F_{s/h}$ <= 1 there is no increasing the performance and in general the S/H should not be added.

$$F_{s/h} = T_{ap(s/h)}/T_{ap(adc)} \quad (5)$$

Where: $T_{ap(s/h)}$ is the aperture time of the sample and hold circuit and $T_{ap(adc)}$ is the aperture time of the converter

F_{s3db} or 3dB sampling frequency is the sampling frequency guarantied maximal error E_{max} less than or equal to 3db. The corresponding equation is $F_{s3db} = 4 F_{max}$. Also is called the **frequency of 3dB modulation.**

F_{s100} is the main (first) frequency of 100% modulation The term is intended to replace "the Nyquist frequency" or the "frequency of exact reconstruction". The corresponding equation is $F_{s100} = 2 F_{max}$. **F_{s100}** is defined with SSF N=2 and with maximal amplitude error between 0 and 100% included.

(Law of the) Maximal amplitude error E_{ssmax} during the conversion of SS (DC and phase components are zeros) is given with the equation below:

$$E_{ssmax}=(1-\sin(90-180/N)=(1-\cos(180/N)) \quad (6)$$

(Law of the) Maximal amplitude error E_{csmax} during the conversion of CS (DC and phase components are zeros) is given with the equation below:

$$E_{csmax}=(1-\cos(90-180/N)=(1-\sin(180/N)) \quad (7)$$

(Law of the) minimal errors during the conversion of SS or CS (DC and phase components are zeros) is stating that SSF

$$N=4*k \quad (8)$$

(k=1,2,3…) is giving the opportunity to obtain zero amplitude, phase and frequency errors during the regular sampling and DC component error is always zero.

Non-reproducible bits (N_{nrb}) – bits which could not be reproduced in a repetitive way, e.g. the bits determining the resolution of (of a converter). N_{nrb} is the difference between the bits (e.g. of the converter) which is determining the resolution N_{res} (or the whole number of bits N_{all}) and the bits determining the accuracy or the reproducible bits (N_{ab}). N_{nrb} cannot be reproduced or predicted in all of the test cases. The equations below are clarifying the definition:

$$N_{nrb} = N_{all} – N_{ab} \quad (9)$$

$$N_{nrb} = N_{res} – N_{ab} \quad (10)$$

One dimensional sampling of SBLS – is the sampling of the signal given with the equation below:

$$X(t)=X_m\sin(\omega_x t+\varphi_x)+X_0 \quad (11)$$

During the one-dimensional sampling at least four samples are needed in order to calculate the four parameters of the signal.

Phase modulation during the sampling process – the change of the amplitude of the samples when the angle of the first sample is changed. (The sampling and the sampled frequencies and the amplitude of the sampled SBLS are constants).

Postulate about basic properties of the real signals – the postulate which is stating that every real signal has the following basic properties: 1. Finite amplitude and peak to peak amplitude. 2. Finite power in every moment and during its existence. 3. Finite spectrum. 4. Finite number of spectral lines. 5. Finite slew rate, first derivative and every other derivative. 6. Could be represented as a finite sum of SBLS. 7. Is a smooth (uninterrupted) function.

Principle "One sample per parameter to reconstruct" is stating that the signals need at lest one sample per parameter for parameters calculation and reconstruction.

Real signal – A signal which could be presented as a finite sum of the simplest band limited signals (SBLS) and with the following properties: 1. Finite slew rate (finite first and every other derivative). 2. Finite number of maximums and minimums. 3. Representing a continuous mathematical function. 4. Finite number of spectral lines 5. Finite energy (power) in each moment and during its finite existence.

Reproducible bits – bits which could be reproduced easily and repeatedly. Thee are determining the accuracy (e.g. of a converter).

Sampling angle θ_s is defined for a SS or CS signal with the equation below:

$$\theta_s = 360/N \quad (12)$$

It is measurable in degrees.

Angle of the maximum deviation from the maximum of the sine or cosine signal is the angle of the maximal amplitude error during the sampling:

$$\theta_{smax} = 180/N \quad (13)$$

It is measurable in degrees.

Sampling theorem for SS is stating that for the SS (with DC and phase components are zeros and SF N>= 2) sampling factor N is given with the equation below when the maximal amplitude error E_{max} is given:

$$N = 180/(90-\arcsin(1-E_{max}) \quad (14)$$

Sampling theorem for CS is stating that for the CS (with DC and phase components are zeros and SF N>= 2) N is given with the equation below when the maximal amplitude error E_{max} is given:

$$N = 180/(90-\arccos(1-E_{max}) \quad (15)$$

Sampling factor (SF) N or "signal sampling factor" (SSF) is given with the equation below

$$N = F_d/F_s =F_{max}/F_s \quad (16)$$

where F_d is the sampling frequency, F_s is the frequency of the sampled sinusoidal or co sinusoidal signal and F_{max} is the maximal frequency of the sampled band limited signal (BLS)

SBLS (the simplest band limited signal) is the simplest signal with two lines into its spectrum – one is a direct current (DC) and the other is a sine or cosine wave. The following two equations are applicable:

$$A = A_m \sin(2\pi f + \theta) + B \qquad (17)$$
$$A = A_m \mathrm{cosin}(2\pi f + \theta) + B \qquad (18)$$

The SBLS is the simplest test signal with two lines into spectrum.

Trapezoidal staircase function with no rounded (ideal) angles – the function used to approximate the reconstructed sampled signal. The finite slew rate is and advantage of that model compared to the rectangular staircase function but the idealized angles is making it too idealized and cannot be approximated with finite sum of SBLS. The trapezoidal function with rounded angles is the better solution.

Trapezoidal function with rounded angles (non interrupted or continuous trapezoidal function) – the only possible presentation of the reconstructed signal before filtering.

The principle of the limited values (finity) of the signal parameters is stating that every real signal has finite values of its parameters e.g. finite slew rate (SR), finite energy, finite spectrum, finite number of spectral lines, etc.

Reproducible bits (N_{rb})– the bits (e.g. of the converter) which is determining the accuracy and which could be reproduced under the testing conditions.

$$N_{rb} = N_{all} - N_{nrb} \qquad (19)$$

Errors of the direct reconstruction – the errors between the corresponding parameter of the input analog signal and the parameter of the reconstructed signal. The method of direct reconstruction with ADC and DAC with the same number of reproducible bits is used as a reference.

The sampling process is defined as process of conversion of an analog signal into staircase function with rounded angles.

The sampling rate Fs is the frequency of the taken and memorizing of the samples. It is related with the number on the parameters to reconstruct (k) and with the number of the spectral lines to reconstruct (p). The following two rules are respected: 1. At least one sample per parameter to reconstruct. 2. At least four samples per alternative current (SS/CS) spectral line. Applying the two rules simultaneously is guaranteeing the exact and predictable signal reconstruction. If we keep the band of the signal constant and if we are increasing the number of the spectral lines we will increase also the number of the samples required to reconstruct the parameters of the spectral lines in the complex signal.

Coefficient of changing of the sampling rate - is a term intended to replace the term "decimation" and an appropriate coefficient of changing should be defined:

$$K_{chs} = F_{is}/F_{os} \quad (20)$$

where F_{is} is the initial sampling frequency and F_{os} is the resulting (output) sampling frequency.

The following terms are much more self explanatory and representative that the terms "total harmonic distortion (THD) and "aliasing":

In band added frequencies – sum of (the energies) of the added frequency components in the signal band due to nonlinear sampling (and reconstruction) process. The sum could be divided to the energy (amplitude, power) of the initial signal.

Out band added frequencies - sum of (the energies) of the added frequency components outside the initial signal band due to nonlinear sampling (and reconstruction) process. The sum could be divided to the energy of the initial signal.

Total added frequencies – sum of (the energies) of all added frequency components to signal due to nonlinear sampling (and reconstruction) process. Could be divided to the energy of the initial signal.

V. Conclusions

Basic terms in the signal sampling theory are misleading and needs of replacement. Some of the basic concepts and models should be reevaluated and replaced with models which are closed to the real signals. Repeating wrong terms does not make them useful and they should be replaced with more accurate terms which are self-explanatory. New terms are proposed and defined.

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