World Federation of National Mathematics Competitions: Why, How and Where to?

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Summary.

The appearance and development of organized Mathematics Competitions is an important 20th century phenomenon. The participation growth was tremendous: for one century, the number of participants grew from few hundreds to tens of millions.

Competitions help identify students with higher abilities (not only in mathematics). They motivate these students to develop their abilities and to seek professional realization in science. Competitions have positive impact on education and on educational institutions. They called to life a great variety of mathematical enrichment activities which gradually embrace all the world.

The World Federation of National Mathematics Competitions (WFNMC) is both a result of this development and an instrument for its acceleration. The Federation is now 22 years old, so it is important to evaluate what has been achieved and what has to be done in order to improve the specific services provided by WFNMC to the international mathematical community. In this address I will try to outline and discuss this topic.

1 Math Competitions in 20th Century

It is difficult to trace precisely the origins of mathematics competitions for school students.

V. Berinde [**?**] reports that a primary school math competition with 70 participants was held in Bucharest, Romania, as early as 1885. Eleven prizes were awarded to 2 girls and 9 boys. It is possible that similar competitions were held elsewhere, even before 1885.

Nevertheless, the 1894 Eötvös competition in Hungary is widely credited as the forerunner of contemporary mathematics (and physics) competitions for secondary school students. The competitors were given four hours to solve three problems (no interaction with other students or teachers was allowed). The problems in the Eötvös competition were designed to check creativity and mathematical thinking, not just acquired technical skills. In particular, the students were often asked to provide a proof for a statement.

As an illustration, here are the three problems given in the very first Eötvös competition in 1894. The entire collection of problems and their solutions is maintained by John Scholes at

`www.kalva.demon.co.uk/eotvos.html`:

P1. Show that $\{(m,n) : 17 \text{ divides } 2m+3n\} = \{(m,n) : 17 \text{ divides } 9m+5n\}$.

P2. Given a circle C, and two points A, B inside it, construct a right-angled triangle PQR with vertices on C and hypotenuse QR such that A lies on the side PQ and B lies on the side PR. For which A, B is this not possible?

P3. A triangle has sides length a, $a+d$, $a+2d$ and area S. Find its sides and angles in terms of d and S. Give numerical answers for $d=1$, $S=6$.

The Eötvös competition model still dominates the competition scene.

The year 1894 is notable also for the birth of the famous math journal *KöMaL* (an acronym of the Hungarian name of the journal, which translates to *High School Mathematics and*

Physics Journal). Founded by Dániel Arany, a high school teacher in Györ, Hungary, the journal was essential to the preparation of students and teachers for competitions (about one third of each issue was devoted to problems and problem solving and readers were asked to send solutions). As noted by G. Berzsenyi in the preface of [**?**], about 120–150 problems were published in *KöMaL* each year; about 2500–3000 solutions were received. The best solutions and the names of their authors were published in following issues. This type of year-round competition helped many young people discover and develop their mathematical abilities; many of them later became world-famous scientists. (For more information, see the journal web site, `komal.elte.hu`.)

About the same time, similar development occurred in Hungary's neighbor, Romania. The first issue of the monthly *Gazeta Matematică*, an important journal for Romanian mathematics, was published in September 1895. The journal organized a competition for school students, which improved in format over the years and eventually gave birth to The National Mathematical Olympiad in Romania. The journal was transformed to *Society Gazeta Matematică* in August 1909. The following year, the Romanian Parliament approved the legal status of the new society and this is considered to be the birthday of the Romanian Mathematical Society [**?**].

What happened in Hungary and Romania in the late 1800's was not something isolated and special to these two countries only; most likely, it reflected a much broader trend. Indeed, the spirit of international collaboration and solidarity was rising steadily and many national math societies were founded around the same time. The Olympic Games were revived in 1896. The First International Congress of Mathematicians took place in Zürich in 1897. Within several decades, other countries started to organize mathematics competitions. In 1934, a Mathematical Olympiad was organized in Leningrad, USSR (now St. Petersburg).

In the middle of the century, the flagship of math competitions, the International Mathematics Olympiad (IMO), was born. The first IMO took place in Romania (1959) with participants from seven countries: Bulgaria, Czechoslovakia, German Democratic Republic, Hungary, Poland, Romania, and the Soviet Union (USSR). The second IMO (1960) was organized by Romania as well, but since then it is hosted by a different country every year (except 1980, when no IMO was held). Over the years, the participation grew dramatically: the 2005 IMO in Mexico gathered 513 competitors from 93 countries! This is the most prestigious math competition today. Directly or indirectly, all other competition activities in mathematics are related to IMO.

The Australian Mathematics Competition (AMC) started in the seventies and soon reached half a million participants. The European competition named Kangaroo (modeled after the AMC), which started in the nineties, today involves 3.5 millions of students from different countries.

A Mathematical Olympiad for public school students was initiated in Brazil only two years ago. It alone now has 12 million participants!

Altogether hundreds of competitions and competition-like events with national, regional, and international importance take place every year. There are "Inclusive" (open for all) competitions which are intended for students of average abilities, while "exclusive" (by invitation only) events target talented students (a prime example of the second type is the IMO and the national Olympiad rounds beyond the first). There are "Multiple-choice" competitions where each problem is supplied with several answers, from which the competitor has to find (or guess, as no justification is required) the correct one. In contrast, "classical style" competitions (like the IMO) require the students to present arguments (proofs) in written form. In "correspondence" competitions, such as those organized by *KöMaL* and *Gazeta Matematică*, the students do not necessarily meet each other, while in "presence" competitions (which

form the majority of math competitions) the participants are gathered together, which is believed to provide "equal rights" to all students. There are even mixed-style competitions, with a presence-style first stage and correspondence-style subsequent stages.

In their preparation for various types of competitions, students and their mentors are helped by a large number of books, journals, and other printed and electronic resources.

It would not be an exaggeration to say that the rise and the development of Mathematics Competitions is one of the characteristic phenomena of 20th century, which deserves to be studied and analyzed.

2 Why is WFNMC needed?

To compete means to compare your abilities with the abilities of others. The broader the base for comparison, the better. This seems to be the driving force behind the natural transition from school competitions to town competitions, to national and to international competitions.

Like any other event with positive social impact, each competition or competition-like event generates a group of people dedicated to it. The group consists of team trainers, problem creators, organizers, and other people involved. Taken together, this group maintains and gives the shape of the event. It determines the current status and the future development of the event. This joint obligation (to keep the event floating) serves as a cohesion factor that gradually transforms the group into a vibrant network where collaboration prevails over rivalry.

Such networks have a great "value-added" effect. Learning from others becomes a major source for improvement of your own work. Unlike electrical networks in physics, where energy is conserved and where nodes with higher potential lose part of their potential to nodes with lower potential, math competition networks tend to increase the potential of all of their nodes.

Typical examples of such networks are those associated with competitions as

- the IMO,
- Le Kangourou Sans Frontiéres [www.mathkang.org],
  the Australian Mathematics Competition
  [www.amt.canberra.edu.au],
- the International Mathematics Tournament of Towns
  [www.amt.canberra.edu.au/imtot.html],
- the Ibero-American Mathematics Olympiad
  [www.campus-oei.org/oim/], and
- the Asian-Pacific Mathematics Olympiad
  [www.cms.math.ca/Competitions/APMO/]

— the list is far too short to enumerate all networks that deserve to be mentioned.

A good math competition journal can also create such a network, which comprises the editorial board, the editors, the frequent authors, and the readers. Famous examples are the journals Kvant (Russia), Crux Mathematicorum (Canada), Mathematics Magazine (USA), and Mathematical Spectrum (UK).

All these networks operate autonomously and independently from each other though many of the problems they face are similar in nature. Advancements in one network are not easily transferred to other networks. This is where a role for WFNMC could be clearly seen:

1. to facilitate communication between the different networks,
2. to identify common problems faced by different networks and to provide a proper framework for discussion and solution of those problems;
3. to help newcomers join one (or more) of the networks.

Therefore the essence of WFNMC is a "Network of Networks".

As a matter of fact, some competition networks are connected to each other because they have common members (people who belong to two or more networks). Such people are of special interest to WFNMC because they, on one hand side, know the situation in some networks and, on the other, can directly realize the goals of the Federation in the respective networks. Through them, the role of the Federation becomes feasible.

3 How does WFNMC function?

The WFNMC was founded in 1984, during the Fifth International Congress of Mathematical Education (ICME5) in Adelaide, Australia, under the initiative of Peter O'Halloran. All the activities of the Federation are supported by the Australian Mathematics Trust.

Since 1996 the Federation has a Constitution (revised in 2004). It adopted a "Policy Statement" which stipulates the scope of activities of interest to the WFNMC (especially those related to Competitions and to Education in Mathematics).

The Federation has a "reserved slot" in the programs of every ICME, the International Congress on Mathematical Education, which takes place every 4 years.

Two years after each ICME, the WFNMC organizes its own Conference. The present one in Cambridge is the fifth in a row. The previous were in Waterloo, Canada (1990); Pravetz, Bulgaria (1994); Zhong Shan, China (1998); Melbourne, Australia (2002).

The journal *Mathematics Competitions*, published by the Federation is a powerful tool for networking people engaged with competitions.

The Federation also has its own award, named after Paul Erdös, which is given to people with outstanding contributions to mathematics competitions and to enrichment activities.

In 1994 the WFNMC became one of the Affiliated Study Groups of the International Commission on Mathematical Instruction (ICMI), which, in turn, is a Commission of the International Mathematical Union (for more information about this Commission see [**?**]). In this way, through WFNMC, the different competition networks are incorporated into the global mathematical community.

Each issue of the ICMI Bulletin contains report about the activities of the Federation.

It should be noted that the global competition network resembles existing networks in other mathematical areas, such as Algebra, Geometry, Analysis, and Differential Equations.

In fact, the competition network covers the classical mathematical area known under the (somewhat misleading) name *Elementary Mathematics*. Like other networks, this one operates and lives through its journals, conferences, workshops and e-mail. Periodical regularity of math competitions however adds to the strength and vitality of math competitions networks since the people meet more often.

Unlike other networks which are engaged only with research, the competition network also facilitates the dissemination of best practices in curriculum development and in the work with talented youngsters. New problem solving techniques, new classes of problems, and new ideas about organizing competitions spread quickly around the world.

We should not forget also that, through this global network, the Elementary Mathematics (which constitutes an important part of our mathematical heritage) is preserved, kept alive and further developed.

The WFNMC provides also a framework and a fruitful environment for the discussion of important issues related to mathematical education, to the work with higher ability students and, last but not least, to its own future.

It makes sense to mention here three other developments that show recent trends in the activities of WFNMC.

In ICME 10 (Copenhagen 2004), the members of WFNMC played a decisive role in the Discussion Group 16 "The role of competitions in mathematics education".

The Federation is a key player in the ongoing ICMI Study 16 "Challenging Mathematics in and beyond the classroom".

Several members of WFNMC participate in a Project MATHEU, financed by the Europen Union, focused on the creation of challenging environment for higher ability students.

4 Where to now (What to do next)?

In my opinion there is no urgent need to change radically or revise substantially the main activities of the Federation. For the 22 years of its existence, the WFNMC achieved significant results in developing a world of math competitions.

Nevertheless, it makes sense to critically and permanently assess separate aspects of what was achieved with the intention to extend the scope of the Federation and to focus it on solving problems that are common to various mathematics competition networks. With this in mind, I would like to mention here a few of these problems, without any pretence that the list is exhaustive or consists of most important problems.

4.1 Teachers and Competitions

It has been noted that in many countries, year after year, some schools consistently "produce" more competition winners than other schools. What is the reason behind this phenomenon? Why are some schools more successful than others?

The reasons may be numerous and fairly different in nature. Very often, however, the prominent success of a particular school can be attributed to the dedicated efforts of a single teacher or a small group of teachers. For these excellent teachers, teaching is a vocation, a mission, and not just means to make both ends meet. Such special teachers are real assets for the school and for the whole country. They possess both the necessary mathematical knowledge and the extraordinary personality needed to identify and motivate for hard work the future winners in competitions.

Such teachers need special care, though. To become such a teacher is not easy. The university training of teachers, as a rule, does not emphasize the work with higher ability students. Special courses on such matters are rarely offered.

Higher scientific ability in this area is acquired very slowly, at the expense of great personal efforts. Even more so, if this happens in the absence of academic support after the university education.

It is no secret that the success of these teachers depends very strongly on their working environment and on the appreciation by their colleagues and administration. Very often however the actual working conditions in the schools do not support the work and the development of these dedicated teachers.

What can be done to improve this situation? For instance, the materials made available to the teachers for the work with higher ability students should not include problems and solutions only, but also provide methodological instructions for the teachers, suggesting how to use these materials and what type of reactions or difficulties to expect on the side of students. For this to happen, more specialized research is needed, conducted with the help of professional math educators.

In fact, many organizations which are involved with competitions organize seminars and workshops for teachers as well. Since many countries already have accumulated valuable experience (both positive and negative) working with such teachers, it makes sense to discuss and evaluate the results and to recommend the good practices for dissemination. Teachers are the most valuable human resource for the development of competitions and related activities.

4.2 Where is the proper home of competition activities?

At many universities, the competition-like activities are not "at home" (and therefore not appreciated) neither in Mathematics Departments (because "competitions concern Elementary Mathematics"), nor in Mathematics Education Departments (because they are "too mathematical and refer to the relatively small group of talented students"). As a result, people involved with competitions are neglected when it comes to promotions and other forms of appreciations. These negative attitudes can be dangerously discouraging to the people involved with competitions.

The WFNMC must do something in order to change these attitudes. It is necessary to draw the attention of Mathematics Departments to the fact that competitions help identify students with higher abilities and motivate them to seek professional realization in science. Colleagues from Math Education Departments must be convinced that competitions have positive impact on education as a whole and on educational institutions. It is time for both communities (research mathematicians and mathematics educators) to understand their joint interest in supporting competitions and the people involved with them.

4.3 Competitions with "open ended" problems

The majority of competitions that exist today cultivate the ability to answer questions and problems posed by *other* people. Nevertheless the ability to *formulate* relevant questions related to a problem or a situation is also important, especially for those going to become researchers.

Here is another observation in the same direction: To be successful in a competition, a student often needs not only a good mind, but a very quick one. Most competitions are limited in time to just 3-4 hours, imposing a significant stress on the nervous system of their participants. Not only do students have to solve the problems correctly, they have to do so quickly and in the presence of their direct competitors. Yet, there are many highly creative students, who do not perform well under pressure. Such "slow thinkers" often come up with new and valuable ideas a mere day (or even just five minutes) after the end of the competition, yet receive no reward or incentive.

Traditional competitions disadvantage such students, even though some of them are highly creative and could become good inventors or scientists. Indeed, what matters in science is

rarely the speed of solving difficult problems posed by other people. More often, what matters is the ability to formulate questions and pose problems, to generate, evaluate, and reject conjectures, to come up with new and non-standard ideas. All these activities require ample thinking time, access to information resources in libraries or the Internet, communication with peers and experts working on similar problems, none of which are allowed in traditional competitions.

Obviously, other types of competitions are needed to identify, encourage, and develop such special "slower" minds. The competitions should reflect the true nature of research, containing a research-like phase, along with an opportunity to present results to peers — precisely as it is in real science.

As a matter of fact, such competitions, designed to identify students with an inclination to scientific (not only mathematical) research, already exist. It only makes sense for WFNMC to promote them widely and vigorously.

4.4 Science trends reflected in Competitions

WFNMC can improve its work by implementing current science trends into competitions and by targeting other audiences of students.

4.4.1 Algorithms in Mathematics

The nature of mathematical research has changed significantly since considerable computing power came to the desk of almost every researcher and student. Mathematicians today can conduct complicated numerical experiments, use software for complex algebraic and analytic transformations, find patterns in huge data sets. Like the experiments in other sciences, this could help reject some conjecture or formulate a new one. Thus, research in mathematics became similar to research in other sciences.

All this is based on mathematical algorithms. Algorithmic thinking is getting higher importance and successfully complements the "axiomatic" approach in mathematics.

This change should be duly reflected in the creation and selection of competition problems. Perhaps more problems should be offered at various competitions where algorithms and their properties are focused in order to cultivate algorithmic thinking. Otherwise, we will become witnesses of a "brain-drain" and the best young minds will be driven to competitions in Informatics.

4.4.2 Teamwork

Working in teams is a well-established trend in modern science. For centuries, research in mathematics has been a solitary endeavor. Today, we see more and more teamwork in mathematics and, especially, in its applications. This reveals yet another similarity between modern mathematics and the other sciences (where teamwork has traditionally deeper roots). The ability to work in a team is valuable skill that could and should be cultivated early on.

Mathematics team competitions could contribute a lot in this direction. There are many such competitions around the world; it only makes sense to make them more popular.

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